

Enrollment No: \_\_\_\_\_

Exam Seat No: \_\_\_\_\_

# C.U.SHAH UNIVERSITY

## Summer Examination-2017

Subject Name: Discrete Mathematics

Subject Code: 4TE04DSM1

Branch: B.Tech (CE,IT)

Semester: 4

Date: 03/05/2017

Time: 02:00 To 05:00

Marks: 70

### Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
  - (2) Instructions written on main answer book are strictly to be obeyed.
  - (3) Draw neat diagrams and figures (if necessary) at right places.
  - (4) Assume suitable data if needed.
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### Q-1 Attempt the following questions:

(14)

- a) Find the least and greatest element in the POSET  $\langle N, D \rangle$ , if they exist. (01)
- b) Define: Equivalence relation, Comparable element. (02)
- c) State Pigeonhole principle. (01)
- d) Prove that  $(ab + ab')a'b' = 0$ . (02)
- e) Find the atom and anti-atom of  $\langle P(X), \subseteq \rangle$ . (01)
- f) State Handshaking theorem and define cycle. (02)
- g)  $(Z_{10}, +_{10})$  is cyclic group.- True or False? (01)
- h) Define: Difference of two fuzzy sets. (02)
- i) In how many ways can the letters of the word MONDAY be arranged? How many of them begin with M and end with Y? (02)

### Attempt any four questions from Q-2 to Q-8

### Q-2 Attempt all questions.

(14)

- a)  $\langle L, *, \oplus \rangle$  is a lattice an algebraic system then there exist an order relation  $\leq$  on L such that  $\langle L, \leq \rangle$  is a lattice as a poset. Where  $a * b = \text{glb}\{a, b\}$ ,  $a \oplus b = \text{lub}\{a, b\}$  for  $\forall a, b \in L$ . (05)
- b) For a lattice  $\langle S_{60}, D \rangle$ , answer the following questions: (05)
  - i) Find cover of each element and draw the Hasse diagram.
  - ii) Find lower bound, upper bound, greatest lower bound, least upper bound of  $A = \{6, 12\}$ .
- c)  $\langle L, *, \oplus \rangle$  is a modular lattice if and only if  $(a * b) \oplus (a * c) = a * (b \oplus (a * c))$ . (04)



**Q-3 Attempt all questions** (14)

- a) Show that  $\{1, 5, 7, 11\}$  is a subgroup of  $(Z_{12}^*, \times_{12})$ , where  $\times_{12}$  is multiplication modulo 12. (05)
- b) Prove that  $\langle P(X), \subseteq \rangle$  is a complemented lattice and also draw the Hasse diagram of it, where  $X = \{1, 2, 3\}$ . (05)
- c) Show that  $w$  is a valid conclusion from the premises  $r \vee s, \sim t \Rightarrow \sim r, \sim s, t \Rightarrow w$ . (04)

**Q-4 Attempt all questions** (14)

- a) Let  $\langle L, *, \oplus, ', 0, 1 \rangle$  be a complemented lattice and for any  $a, b \in L$  then prove that  $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$ . (05)
- b) Let  $E = \{0, 1, 2, 3\}$ ,  $\underline{A} = \{(0, 0.2), (1, 0.4), (2, 0.6), (3, 0.7)\}$ ,  $\underline{B} = \{(0, 0.4), (1, 0.6), (2, 0.8), (3, 0.8)\}$  then find the following: (05)
  - 1)  $\underline{A} \cup \underline{B}$     2)  $\underline{A} \cdot \underline{B}$     3)  $\underline{A} \hat{+} \underline{B}$     4)  $\underline{A} - \underline{B}$     5)  $\underline{A}'$
- c) Obtain the sum of product canonical form of the Boolean expression in three variables  $\alpha(x, y, z) = (x + y)(x + z') + y + z'$ . (04)

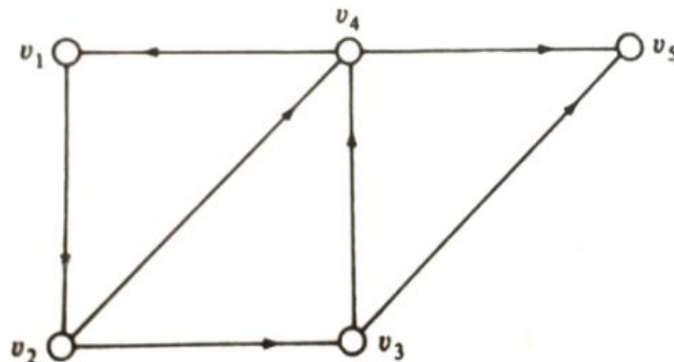
**Q-5 Attempt all questions** (14)

- a) State and prove Stone's representation theorem. (10)
- b) State Distributive law for fuzzy subsets and prove any one. (04)

**Q-6 Attempt all questions** (14)

- a) i) Draw the graph represented by given adjacency matrix  $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$ . (05)

ii) Write the adjacency matrix from the given digraph.



- b) Show that the set  $Q \setminus \{1\}$  is an abelian group with respect to the binary operation  $a * b = a + b - ab$ , for all  $a, b \in G$ . (05)



c) Solve the recurrence relation  $a_n = 5a_{n-1} - 6a_{n-2}$ ,  $n \geq 2$ ;  $a_0 = 1$ ,  $a_1 = 2$ . (04)

**Q-7 Attempt all questions.** (14)

a) Define: Pseudo graph, Cycle, Reachable set, Node base, Level of vertex (05)

b) Prove that  $(Z_5, +_5)$  is a group. Moreover check that it is cyclic or not, if it is cyclic then find generators. (05)

c) By using mathematical induction prove that  $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ . (04)

**Q-8 Attempt all questions.** (14)

a) Draw a directed tree from following and also find the representation of binary tree. (05)

$$\left(v_0 \left(v_1 \left(v_2 \left(v_3 \left(v_4 \left(v_5 \left(v_6 \right)\right)\right)\right)\right)\right)\right) \left(v_7 \left(v_8 \left(v_9 \left(v_{10} \left(v_{11} \right)\right)\right)\right)\right) \left(v_{12} \right)$$

b) State and prove the Lagrange's theorem. (05)

c) Do as directed: (04)

1) Translate each of the statement into symbols, using quantifiers, predicate symbols.

i) "Every student either can speak Tamil or knows C++."

ii) "Some men are genius."

2) Translate the following in your own words.

$A(x)$ :  $x$  is a whale,  $B(x)$ :  $x$  is a fish,  $C(x)$ :  $x$  lives in water.

i)  $(\exists x)(B(x) \wedge \sim A(x))$

ii)  $(\forall x)(A(x) \vee C(x)) \Rightarrow B(x)$

